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Antwoorden midterm toets Kwantumfysica 1
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T1

a) $\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \Psi^*(x)\, \hat{x}\, \Psi(x)\, dx = \int_{-4}^{-2} \frac{2}{10} x\, dx + \int_{2}^{4} \frac{3}{10} x\, dx$

$= \frac{2}{10}\left[\frac{1}{2}x^2\right]_{-4}^{-2} + \frac{3}{10}\left[\frac{1}{2}x^2\right]_{2}^{4} = \frac{6}{10} \quad = \underline{\underline{0.6\ \text{nm}}}$ ← unit

b) $\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$

$\langle \hat{x}^2 \rangle = \int_{-\infty}^{\infty} \Psi^*(x)\, \hat{x}^2\, \Psi(x)\, dx = \frac{2}{10}\left[\frac{1}{3}x^3\right]_{-4}^{-2} + \frac{3}{10}\left[\frac{1}{3}x^3\right]_{2}^{4}$

$= \frac{1}{2}\left(\frac{56}{3}\right) = \frac{56}{6}\ \underline{\underline{\text{nm}^2}}$ ← unit

$\Delta x = \sqrt{\frac{56}{6} - \left(\frac{6}{10}\right)^2} = \sqrt{\frac{5600}{600} - \frac{216}{600}}$

$= \sqrt{\frac{5384}{600}}\ \underline{\underline{\text{nm}}}$ ← unit

c) $P(\langle \hat{x} \rangle - 0.1 < x < \langle \hat{x} \rangle + 0.1) = P(0.5_{\text{nm}} < x < 0.7\ \text{nm})$

$= \int_{0.5\ \text{nm}}^{0.7\ \text{nm}} \Psi^*(x)\, \Psi(x)\, dx = 0$

d) $P(-4\ \text{nm} < x < -3\ \text{nm}) = \int_{-4\ \text{nm}}^{-3\ \text{nm}} \Psi^*(x)\Psi(x)\, dx = \int_{-4\ \text{nm}}^{-3\ \text{nm}} \frac{2}{10}\, dx \cdot \frac{1}{\text{nm}}$

$= \frac{2}{10} = \frac{1}{5}$

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Note: $\langle \varphi_1|\varphi_1\rangle = 1$
$\langle \varphi_2|\varphi_2\rangle = 1$
$\langle \varphi_1|\varphi_2\rangle = \langle \varphi_2|\varphi_1\rangle = 0$

T2

a) $\langle \Psi_0|\Psi_0\rangle = 1 \Rightarrow \left(\sqrt{c_1}^*\langle\varphi_1| + \sqrt{\tfrac{2}{3}}\langle\varphi_2|\right)\left(\sqrt{c_1}|\varphi_1\rangle + \sqrt{\tfrac{2}{3}}|\varphi_2\rangle\right)$

$= c_1\langle\varphi_1|\varphi_1\rangle + \frac{2}{3}\langle\varphi_2|\varphi_2\rangle + \sqrt{c_1}^*\langle\varphi_1|\varphi_2\rangle\sqrt{\tfrac{2}{3}} + \sqrt{\tfrac{2}{3}}\sqrt{c_1}\langle\varphi_2|\varphi_1\rangle = 1 \Rightarrow$

$c_1 + \frac{2}{3} = 1 \Rightarrow c_1 = \frac{1}{3}$

b) $\langle \hat{A} \rangle = \langle \Psi_0|\hat{A}|\Psi_0\rangle = c_1\langle\varphi_1|\hat{A}|\varphi_1\rangle + \frac{2}{3}\langle\varphi_2|\hat{A}|\varphi_2\rangle$
$+ 2\sqrt{c_1}\sqrt{\tfrac{2}{3}}\langle\varphi_1|A|\varphi_2\rangle$

$= \frac{1}{3}A_0 + \frac{2}{3}\cdot 2A_0 + \frac{2\sqrt{2}}{3}\cdot 8A_0 = \frac{5+16\sqrt{2}}{3}\cdot A_0$

c) $|\Psi_M\rangle = |\varphi_1\rangle$

d) and e) $|\Psi(t)\rangle = \hat{U}|\Psi_0\rangle = e^{-\frac{i\hat{H}t}{\hbar}}|\Psi_0\rangle$

$\langle\Psi(t)| = \langle\Psi_0|\hat{U}^+ = \langle\Psi_0|\, e^{+\frac{i\hat{H}t}{\hbar}}$

$\langle \hat{A} \rangle(t) = \langle\Psi(t)|\hat{A}|\Psi(t)\rangle = \langle\Psi_0|\hat{U}^+\hat{A}\hat{U}|\Psi_0\rangle =$

$\left(\sqrt{\tfrac{1}{3}}e^{+\frac{iE_1t}{\hbar}}\langle\varphi_1| + \sqrt{\tfrac{2}{3}}e^{+\frac{iE_2t}{\hbar}}\langle\varphi_2|\right)\hat{A}\left(\sqrt{\tfrac{1}{3}}e^{-\frac{iE_1t}{\hbar}}|\varphi_1\rangle + \sqrt{\tfrac{2}{3}}e^{-\frac{iE_2t}{\hbar}}|\varphi_2\rangle\right) =$

$\frac{1}{3}\langle\varphi_1|\hat{A}|\varphi_1\rangle + \frac{2}{3}\langle\varphi_2|\hat{A}|\varphi_2\rangle + \frac{\sqrt{2}}{3}e^{i\frac{E_2-E_1}{\hbar}t}\langle\varphi_2|\hat{A}|\varphi_1\rangle + \frac{\sqrt{2}}{3}e^{i\frac{E_1-E_2}{\hbar}t}\langle\varphi_1|\hat{A}|\varphi_2\rangle =$

$= \frac{5}{3}A_0 + \frac{16\sqrt{2}}{3}\cos\left(\frac{E_2-E_1}{\hbar}\cdot t\right)\cdot A_0 \quad \Rightarrow$

Amplitude is $\frac{16\sqrt{2}}{3}A_0$

Frequency: $\left(\frac{E_2-E_1}{\hbar}\cdot t\right) = \omega t = 2\pi\cdot f\cdot t \Rightarrow$

$f = \frac{E_2-E_1}{2\pi\hbar} = 3\ \text{GHz} - 1\ \text{GHz}$
$= 2\ \text{GHz}$